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Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2017

Fifth Semester

Core Course—MATHEMATICAL ANALYSIS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

[2013 Admission onwards]

Time : Three Hours

Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Give examples of sets which are :
   (i) Bounded.
   (ii) Bounded below but not above.
2. Define Supremum and infimum of a set S.
3. Define interior point of a set S.
4. Find the limit point of the set $\left\{\frac{1}{n}; n \in \mathrm{N}\right\}$.
5. Define limit inferior and superior.
6. State Cauchy's First theorem on limits.
7. Give an example for Cauchy sequence.
8. Define deleted neighbourhood of a complex number z_0.
9. Reduce $\frac{1+2i}{3-4i}+\frac{2-i}{5i}$ into a real number.
10. Find the value of $\left|e^{i\theta}\right|$.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Show that $|x-y| \geq \|x|-|y\|$ for all $x, y \in \mathrm{R}$.
12. Prove that $r^2 = 2$ has no rational solution.

Turn over

13. If M and N are neighbourhoods of a point x, then show that $M \cap N$ is also a neighbourhood of x.
14. Prove that the set R of real numbers is uncountable.
15. Show that every open interval is an open set.
16. Define convergence of a sequence and give two examples for convergent sequence.
17. If $\{a_n\}, \{b_n\}$ be two sequences such that $\lim a_n = a$ and $\lim b_n = b$, then show that $\lim (a_n \pm b_n) = a \pm b$.
18. Show that $\lim_{n\to\infty} \frac{(5n+1)(n-2)}{n(n+5)} = 5$.
19. Illustrate by an example that every bounded sequence may not be convergent.
20. Show that for any real number x, $\lim \frac{x^n}{n!} = 0$.
21. Prove that inequality $|z_1 + z_2| \geq \| z_1 | - | z_2 \|$.
22. Find all the values of $(-8i)^{1/3}$.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. State and prove Archimedean property of real numbers.
24. Prove the equivalence of Dedekind's property and the order completeness property of R.
25. Give two examples for each of the following :
    (i) Open sets.
    (ii) Closed sets.
26. Prove that a set is closed if its compliment is open.
27. The union of two closed sets is a closed set.
28. (i) If S and T are subsets of R, then show that $(S \cap T)' \subseteq S' \cap T'$.
    (ii) Give an example to show that $(S \cap T)'$ and $S' \cap T'$ are may not be equal.
29. Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$.
30. Show that the sequence $\{S_n\}$ where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ cannot converges.
31. Establish the results $|z^2| = |z|^2$ and $|z^3| = |z|^3$ and hence prove that $|z^3 + 3z^2 - 2z + 1| \leq 25$ if $|z| < 2$.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (i) Show that the interior of a set is an open set.

    (ii) Show that the interior of a set S contains every open subset of S.

33. (i) Prove that every infinite bounded set has a limit point.

    (ii) Prove that the derived set of a set is closed.

34. (i) Prove that every bounded sequence has a limit point.

    (ii) Prove that the set of the limit points of a bounded sequence has the greatest and least members.

35. (i) State and prove Sandwich theorem.

    (ii) Show that the sequence $\{b_n\}$, where $b_n = \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(2n)^2}\right]$ converges to zero.

$(2 \times 15 = 30)$